

## Thermal reliability of Power Modules Using Probabilistic Approaches

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*ABSTRACT. The weak point for the standard power IGBT modules in terms of reliability is thermal fatigue in solder joints due to the thermal stress induced by constitutive materials with different coefficients of thermal expansion (CTE). So far, many researches are aimed at defining accurate finite element simulation with constitutive equations of materials behaviour and fatigue failure relation connecting the inelastic strain and the number of cycles before failure. The aim of this paper is to estimate the probability of failure of power module with the structural reliability methods which taking into account the scatter of input data. Thus thermal material and loading variables are considered as random variables and a thermal failure mode is modelled with the called limit state function. The sensitivities of the probability of failure with respect to the mean and the standard deviation for each random variable have been evaluated.*

*KEYWORDS: IGBT, power module, reliability, FORM, bump, Sn/Ag solder.*

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## 1. Introduction

The natural variability, or uncertainties, existing in reality like geometric dimensions or material properties are not taken into account in finite element modelization. If this variability does not influence the reliability of the system, it is not necessary to consider them in the model. However, if we want to study the sensitivity of reliability with respect to this variability, mathematic method must be associated with traditional finite element modelization. Probabilistic methods can answer this necessity by incorporating these uncertainties in finite element study. This can be done on numerical analysis where thermo mechanical phenomena understanding is essential in the evaluation of the design reliability of power semiconductor modules. One of the weak points for these modules in terms of reliability is the wire-bonding connection, which is here replaced by a solution stemming from the flip chip technology.

This paper deals with the thermal failure mode prediction. To achieve this prediction, this study is divided into three steps: the first one consists in presenting the probabilistic reliability problem and the various methods to resolve this problem with the mechanical-reliability coupling. The second step consists in considering the failure of a power module due to excessive temperature in the chip. Thermal random variables characterizing uncertainties are defined. The last step shows the coupling between the deterministic thermal problem and the probabilistic methods : the random variables become input of the finite element model and a characteristic failure function is defined to take into consideration an excessive temperature in the chip.

## 2. Probabilistic design method

The probabilistic structural approach consists in determining, with a mathematical model, the probability of failure of a given system [Rackwitz, 2001]. Indeed, when we build a traditional finite element model, all the model's input data are considered as fixed values and don't take into account the natural scatter of the parameters. The solution to consider the natural variability and uncertainties of the input parameters on the model is to treat the input data as random variables defined by a law and its associated parameters. All relevant uncertainties influencing the probability of failure are then introduced in the vector  $\mathbf{X}$  of basic random variables. In addition, the failure of the system is modelled by a functional relation  $G(\mathbf{X})$ , called limit state function, and defined to take a null or negative value in the failure domain:

$$\mathbf{G}(\mathbf{X}) \leq 0, \mathbf{X} \in \text{failure domain} \quad (1)$$

The components  $x_k, 1 \leq k \leq n$  of the vector  $\mathbf{X}$  are one realization of basic random variables. It's thus possible to define the probability of failure of the system as:

$$P_f = \int_{G(X) \leq 0} f_X(x) dx_1 \dots dx_n \quad (2)$$

where  $f_X(x)$  is the  $n$ -dimensional probability density of basic random variable vector  $X$ . The problem to solve this integral comes from the limit state function which is not explicit because its evaluation is the result of finite element call. Approximation method can be established to compute the multi-dimensional integral of eq. (Erreur ! Source du renvoi introuvable.) by substituting the limit state function for a linear or second order hyper plane called respectively First Order and Second Order Reliability Method (FORM and SORM)(fig. 1).

The calculation scheme of approximation method is as follow and the code to achieve this task was inspired by Ferum [Haukaas, 2003].

### 2.1 Approximation method

Approximation methods can be established to compute the multi-dimensional integrate in eq. (Erreur ! Source du renvoi introuvable.) by substituting in the standardized Gaussian space the limit state function by a linear or a second order hyperplane, called respectively First Order and Second Order Reliability Method (FORM and SORM) (fig 1).

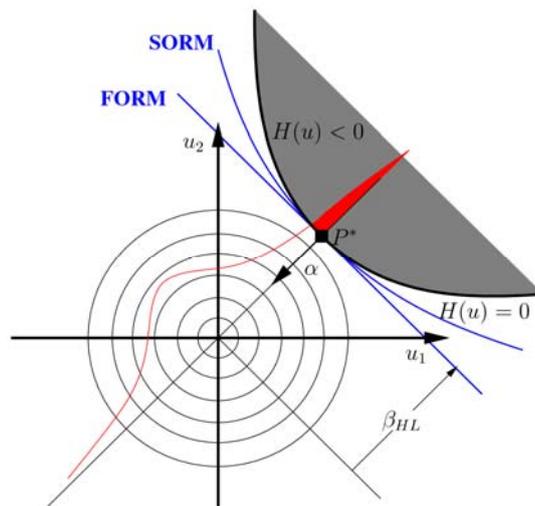


Figure 1: Approximation methods FORM/SORM.

The calculation scheme of approximation method is as follow:

- to obtain a probability of failure independent from the different ways of writing the same limit state function, Hasofer-Lind suggest to map the basic random variables from physical space to independent standardized Gaussian space (Gaussian variable with null mean and unity standard deviation) [Hasofer, 1964]. The limit state function is also mapped in standardized space.
- the most probable failure point, i.e the nearest point to the origin belonging to the limit state in standardized space, is computed by optimization algorithm.
- an approximation of limit state function is built with a hyper plane or a quadratic surface to the most probable failure point to compute the probability of failure.

### 2.1.1 Transformation in standardized Gaussian space

The probability density  $f(X)$  of each random variable  $x_i$  is thus transformed into a density of standardized Gaussian random variable  $u_i$  (Gaussian random variables with null mean and unity standard deviation):

$$\mathbf{u}_i = T(\mathbf{x}_j) \text{ and } H(\mathbf{u}_i) = G(T^{-1}(\mathbf{u}_j)) \quad (3)$$

where

$$f_U(U) = \phi_n(U) = \frac{1}{(2\pi)^{n/2}} \exp\left(-\frac{1}{2}U^T U\right) \quad (4)$$

### 2.1.2 Optimization problem

To find the most probable failure point in standardized space, we must find the nearest point to the origin under the constraint of belonging to the failure state, which is mathematically written:

$$\mathbf{u}^* = \min \{ \|\mathbf{u}\| \mid G(\mathbf{u}) \leq 0 \} \quad (5)$$

The so-called HLRF algorithm, developed by Hasofer and Lind [Hasofer, 1964] and extended to non-normal random variables by Rackwitz and Fiessler [Rackwitz et al., 1978] is used in this study to solve the constrained optimization problem in equation (5)

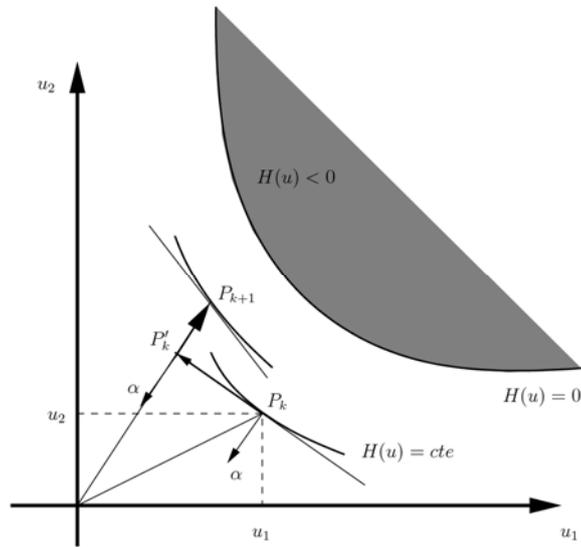


Figure 2: *HLRF Algorithm.*

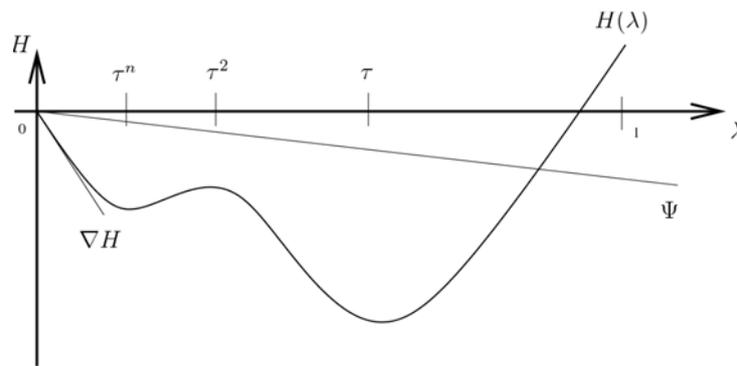


Figure 3: *Armijo rules for iHLRF.*

The algorithm consists in building a sequence of points and finding the next point  $P_{n+1}$  of the iteration as shown in the figure 2 with :

$$P_{n+1} = P_n + d$$

(6)

where  $d$ , the search direction, is computed with:

$$d = \left( \frac{G}{\|\nabla G\|} + \alpha u \right) \alpha^T - u \quad (7)$$

The main problem with HLRF algorithm is that this algorithm is unstable and the convergence is not proved. Liu, Zhang and Der Kiureghian [Liu et al., 1991] improved this algorithm by optimizing the line search scheme and noted iHLRF:

$$u_{n+1} = u_n + \lambda d \quad (8)$$

where the step size  $\lambda$  should be selected to minimize the merit function :

$$merit = \frac{1}{2} \|u\|^2 + c|G| \quad (9)$$

where  $c$  is a penalty parameter chosen to satisfy the condition  $c = \|u\|/\|\nabla G\|$  at each step. Zhang and Der Kiureghian [Zhang et al., 1997] demonstrated, for continuous differentiable  $G(x)$ , the convergence of the iHLRF algorithm. In practice, minimization of the merit function is time-consuming and  $\lambda$  is selected with Armijo's rule according to:

$$\lambda \leq \tau^k \quad (10)$$

where  $0 \leq \tau \leq 1$  is a user-defined parameter and  $k$  is an integer starting with the initial value 0 and increasing by unit at each step until the value  $\lambda$  is less than the initial merit function, as illustrated in figure 3.

This efficient search algorithm requires the computation of the gradient vector  $\nabla G(x)$  to find the next point (eq. (Erreur ! Source du renvoi introuvable.)). The differentiation of the limit state function is given by:

$$(11) \quad \nabla G(x) = \frac{\partial G(x)}{\partial x} \simeq \frac{G(x+h) - G(x)}{h}$$

where  $\nabla G(x)$  is the gradient of the limit state function. For this study, the gradient is computed with the Forward Finite Difference Method (FDM) as described in equation (Erreur ! Source du renvoi introuvable.).

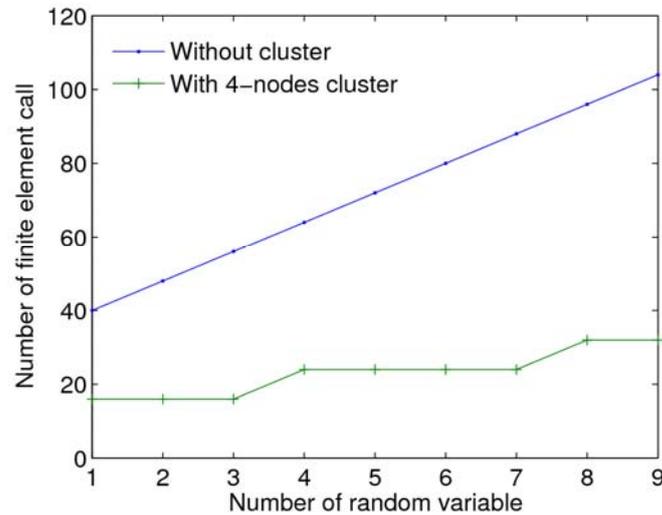


Figure 5: Time to compute solution without cluster and with a 4-nodes cluster.

### 2.1.3 Probability of failure and sensitivities

Hence, in finite element reliability analysis, the main effort in evaluating this gradient is devoted to computing the centred value of  $G$ , the different values around this point for each random variable increased by the spacing value  $h$  and the step size  $d$ . The computational effort is reduced by a parallelization method with the use of a cluster with MPI (Message Passing Interface) protocol. Each gradient evaluation with respect to random variables is achieved by one or several processors according to the figure 4: One processor computes the limit state function with the initial values of the vector  $X$  while at the same time, the others compute the forwarded points. After finding the direction with the gradient method, the computing of the step size  $d$  can be parallelized too : several merit functions are computed at the same time to calculate a number of merit function value equal to the number of processors available. As a result, in figure 5, the evaluation of the limit state function with these different gradients does not take more time computing for a number of random variables less or equal to the number of processors.

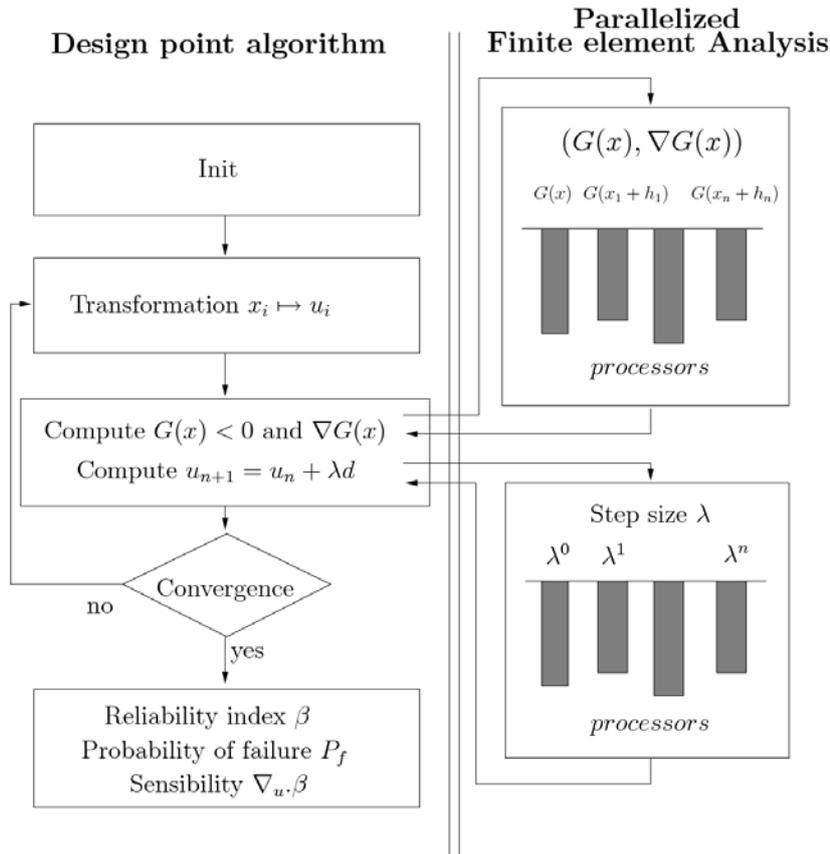


Figure 4: Design point optimization algorithm.

The first order approximation of the probability of failure  $P_f$  with the half space fitted to the true failure domain at the design point can be written with the relation:

$$P_F \approx \Phi(-\beta_{HL}) \tag{12}$$

where  $\beta_{HL}$  is the reliability index following the Hasofer-Lind definition, i.e the distance between origin and the design point.

The main results available with approximation methods is then the probability of failure of the system (eq. 3) but other interesting results are the sensitivities of this probability regarding the different basic random variables [Lemaire, 2005]. In FORM formulation, the limit state function can be written  $H(u)=au+\beta$  under normalized form. We can then define the sensitivity of the reliability index with respect to the standardized random variables as follows:

$$\alpha = \nabla_u \beta = \left[ \frac{\partial \beta}{\partial u_1}, \frac{\partial \beta}{\partial u_2}, \dots, \frac{\partial \beta}{\partial u_n} \right] = \frac{\nabla_u H(u)}{\|\nabla_u H(u)\|} \Big|_{u^*} \quad (13)$$

We can study the sensitivity  $\alpha_{p_i}$  of the reliability index  $\beta$ , and then the probability of failure, with respect to the distribution parameters, such as mean or standard deviation, of the  $\gamma^{th}$  random variables to identify those which must have stricter quality control. The sensitivity  $\alpha_{p_i}$  is calculated with:

$$\nabla_p \beta = -J_{u^*,p} \alpha \quad (14)$$

where  $J_{u^*,p}(i,j) = \frac{\partial T_1(x^*,p)}{\partial p_j}$  is the Jacobian matrix between the Hasofer-Lind transformation and the parameter p.

This sensitivity cannot be used directly for comparison, so the elasticity is defined by the normalisation of this sensitivity:

$$e_{p_i} = \frac{p_i}{\beta} \alpha_{i,r} \text{ with } \alpha_{i,r} = \frac{\partial \beta}{\partial p_{i,r}} \Big|_{u^*} \quad (15)$$

### 3 Modelling methodology

The package modelled in this study is a new IGBT power module where the IGBT connections are derived from the flip-chip technologies. To evaluate the reliability of this package with the method presented above, we need to characterize the constitutive behavior laws of the different materials with the uncertainties on the different coefficients and to define the failure function depending on the failure mode.

### 3.1 Geometry model

The numerical model represents the specimen subjected to creep test. The eutectic Sn/Ag solder is used to joint two substrates with cylinder-shaped bumps (1.45 mm diameter and 2 mm long) and the IGBT chip. Fig 6 shows the assembly with bump technology. The bump connection is soldered to the IGBT with a gold layer, and to the substrate with 250 m aluminium metallization. For the development of reliability investigation, a very particular attention must be given to the finite element model to reduce the time spent computing. Considering the particular geometry, only a quarter of the entire multi-layer switch was numerically modelled.

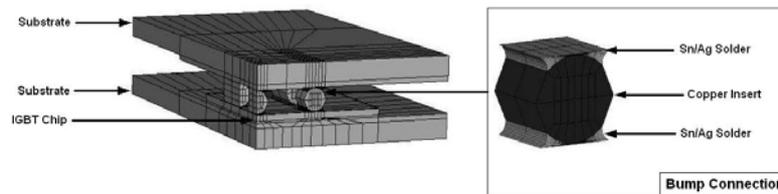


Figure 6: *The 3-D finite element model.*

### 3.2 Thermal loading history

The power IGBT module is subjected to thermal cycling loading. We can consider two types of loading according to the type of test: the passive cycling test and the active power cycling test. During the passive cycling test, a uniform temperature cycle can easily be modelled by applying the same thermal history loading on the entire model. The active power cycling is more difficult to compute because of a preliminary analysis of heat transfer. For this power cycling, temperature time history loading is only imposed on the IGBT element. To simulate the water cooling, a coefficient of heat exchange is applied on the lower and the upper surfaces of the multi-layer module and the temperature of the water is fixed at the value of 70 °C. A best definition of temperature boundary condition will have to be made to find an accurate solution [Tawashiraporn et al., 2002] : incorrect boundary conditions are expected to lead to incorrect temperature profile, incorrect resultant stress and then false location and magnitude failure. The time history loading amplitude is shown in figure 7 and the temperature map result in figure 8.

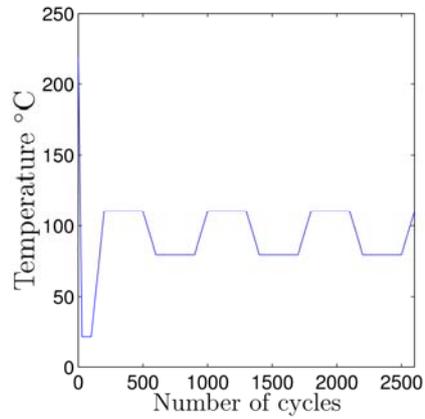


Figure 7: Thermal load history.

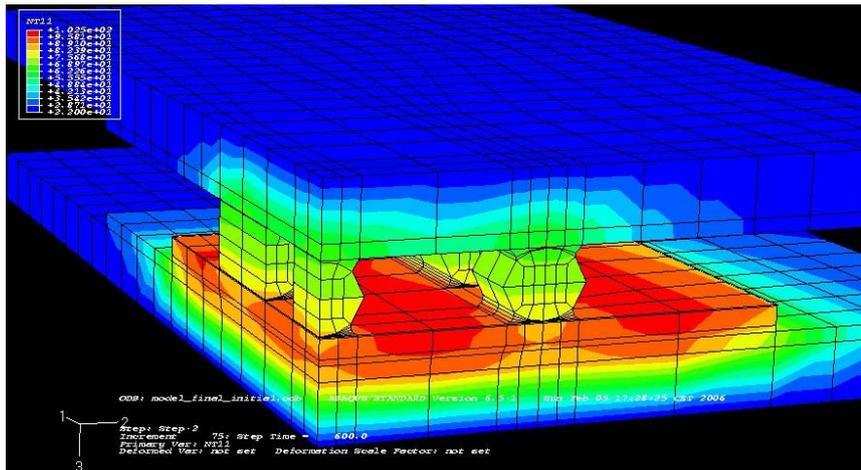


Figure 8: Temperature map.

### 3.3 Thermal failure mode

The limit state function represents here the failure mode where the IGBT temperature exceeds a nominal fixed temperature.

The system falls in the failure domain if the temperature of any silicon chip nodes exceeds the limit value:

$$G(X) = T_{max} - T_{max_{chip}} \quad (16)$$

with

$$T_{max_{chip}} = \max(T_{nodes_i} | i \in \text{chip}) \quad (17)$$

The deterministic computing (i.e. all the values of the different random variables are taken as their mean values) of this module reports that the  $T_{max}$  is equal to 122 C. We consider here the conductivity of the AlN substrate, the imposed heat flux of the chip, the coefficient of the thermal transfer  $h$  between the substrate and the cooling water and the cooling water temperature as random variables with a mean and a standard deviation as represented in table 1.

Random variables	type of distribution	Mean	standard deviation
Heat Flux ( $\text{W}/\text{cm}^2$ )	normal	200.	2.
Water temperature( $^{\circ}\text{C}$ )	normal	70	10
Coefficient of heat transfer $h$ ( $\text{W}/\text{m}^2$ )	log-normal	5000	500
AlN conductivity ( $\text{W}/\text{m.K}$ )	log-normal	30.	3.

Table 1: Random variables for thermal analysis.

### 3.4 Results

The iHLRF algorithm has a good convergence : in fact, the heat transfer analysis is a linear problem and the limit state function is linear too : with an exact evaluation of the limit state function, the iHLRF optimization algorithm with gradient method can find the exact minimum. Here, even with the error due to the finite element approximation and a slightly non-linear transformation in standardized Gaussian space, the final algorithm point remains in the tolerance of the stop algorithm criteria : the algorithm takes 2 iterations to achieve the solution, each one of these iterations having a step size equal to 1, i.e the maximum of the possible step size, showing then the speed of the algorithm convergence. For an algorithm which is not parallelized, this analysis can be made with 3 iterations, which take 5 computes for the limit state and its gradient, plus 2 evaluations of the step size, which give 17 finite element successive calls. Here, the same analysis is made with a time corresponding to 5 finite element calls (three for the function and its gradient and two for the step size). The advantage of the parallelization method is not completely shown here since the time to compute the step size is the same in the two methods.

In this configuration, the reliability index  $\beta$  is equal to 2.39 ( which is equivalent to a probability of failure  $P_f=8.27.10^{-3}$ ) with the fixed upper temperature . Figures 9 and 10 show

the different elasticities of random variables with respect to the mean and the standard deviation. The conductivity of the substrate does not have a great influence on the finite element output temperature. The water temperature and the imposed flux influence the most this thermal failure mode. It is interesting for future developments, where thermal and mechanical analysis will be coupled, to see that the mean of chip flux value takes an important part in the probability of failure even if this standard deviation is small.

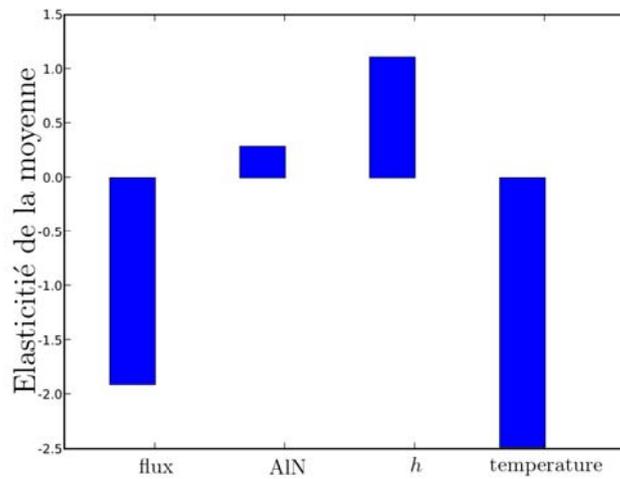


Figure 9: Elasticities of  $\beta$  with respect to the mean.

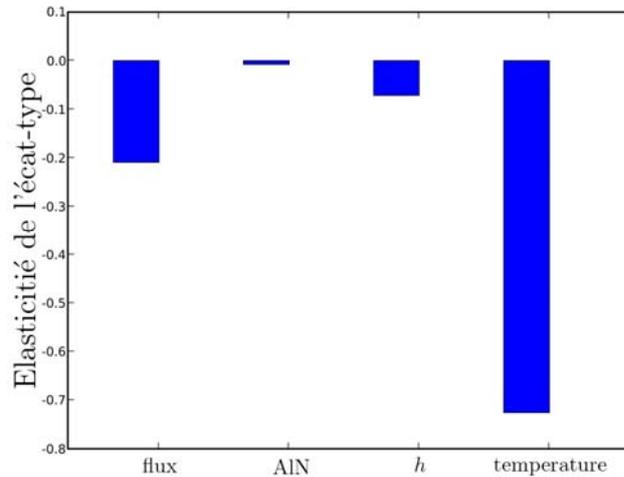


Figure 10: *Elasticities of  $\beta$  with respect to the standard deviation.*

#### 4. Conclusion

In this article, we attempt to estimate the reliability of the power module connections with structural reliability method.

The uncertainties on the material variables can be modelled as random variables and we can define a limit state function which is able to modelize the failure. Probabilistic approximation methods are used to define the probability of failure and its sensitivities according to the random variables.

A study is made to evaluate reliability with a thermal failure. The iHLRF algorithm shows good convergence and demonstrates the interest of the parallelized algorithm. The reliability analysis was made by taking different materials characteristics and loading as random variables. It shows the importance of the thermal loading with the imposed flux and the temperature of water cooling. These imposed parameters not being adjustable in the design stage, the effort to increase the reliability will have to be made on the thermal coefficient exchange  $h$  by choosing appropriate materials. Thermal behaviour is not sufficient to understand the failure of power modules and another study must be made to consider the mechanical failure mode with the fatigue of the solder joints.

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