
Modelling of reliability, of mechanical components submitted to various operating conditions via a parametric model using Cox's proportional hazard rate

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Abstract: Within the framework of the data processing in exploitation, it is necessary to propose a model, which can be used to do the various traditional inferences of reliability. Very often the exponential law has been used for its simplicity. However, this model is not very representative of the lifetimes of mechanical components and does not take into account the various environmental conditions. The main objective of this work, is to complete classical models with a parametric function (Cox's model). This method is developed and compared with the simplified models.

KEYWORDS: Reliability mechanical, Cox's model, parametric models.

1. Introduction

In this paper is proposed a new approach to modelize the reliability of mechanical components. The model is built from processing data reflecting the various operating conditions of the components. The main originality of our approach is that it incorporates a parametric Cox's model which is used to take into account the various operational conditions. This method is described and compared in the context of a simulated application.

Notations

$\lambda_0(t)$: basic failure rate

$\lambda(t,z)$: failure rate with operating conditions characterized by the vector Z

$R_0(t)$: basic reliability law

$R(t,z)$: reliability law with operating conditions characterized by the vector Z

λ : parameter of the exponential law

β, η, γ : parameters of the de Weibull law

2. Principle of the method

The representation of the reliability is generally established from the knowledge of the of failures rate, the rate itself is related to various parts of the bathtub curve. The simplest model will have only a single parameter (exponential law for example). We should expect that the quality of fitness increases with the number of parameters. For mechanical components, the Weibull's model with 3 parameters is more powerful than a simple exponential law, [LYO 06], [97 BIR]. However, this model alone does not take into account the different environmental conditions that characterize the behaviour of existing mechanical components. We propose to use a complementary function to take into account the environmental characteristics. The increase of the number of parameters must improve the model, this is what we want to check.

2.1. Addition of a Cox's model

When the component or material is used in conditions which differ from those so-called normal, the rate of failure is changed. In the same way, different conditions both in the manufacturing process and design will lead to variations of this rate. All these changes can be incorporated into a Cox's model [COX 84], [82 AND], this feature is very useful for improving the estimation of the reliability of components, this leads us to choose the model rate of failures:

$$\lambda(t, z) = \lambda_0(t) e^{B^T \cdot Z}$$

B, Z : are two vectors of \mathbb{R}^k ,

$B^T \cdot Z$: Represents their inner product, the vector Z corresponds to the environmental conditions of the component which are known, B is deterministic, generally unknown, it represents the influence of the manufacturing process, design features, service conditions, components of this vector yet to be estimated.

The vector $Z = [z_1 \ z_2 \dots \ z_k]$ corresponds to the different operating characteristics, manufacturing and design of the component. At each value z_i ($i = 1, \dots, k$), is associated a characteristic noted $z_i^{n_i}$, where n_i represents the number of characteristics attached to the component number i . These features are coded, which allows the introduction of qualitative data. In general, it is associated with the value 0 the basic conditions (or normal) operating, for these conditions we will find the failure rate base.

$$\lambda(t, z_0) = \lambda_0(t) e^{B^T \cdot Z_0} = \lambda_0(t) \text{ with } Z_0 = [0, \dots, 0]$$

For other operating conditions, the coding is suitable, we can use 1, in the case of two conditions, thus z_1 takes the value 0 or 1 (which is denoted by $z_1^1 = 0$, et $z_1^2 = 1$). For more than two terms, the coding is different; z_1 may be set to 0 or 1, 2 or 3. (which is denoted $z_1^1 = 0, z_1^2 = 1, z_1^3 = 2, z_1^4 = 3 \dots$). The problem of coding is a delicate point, the use of nested classes can answer some cases, this has already been the subject of an abundant literature and will not be developed here.

2.2. Complete Model

The model representing the failure rate of a component with its environmental conditions has a base rate $\lambda_0(t)$ and an environmental function, which are to be determined. The choice of the model of the base rate can be achieved in all generic models used in reliability. One can choose an exponential model for electronic components and Weibull model for mechanical components, without excluding the other possibilities (Log-Normal, Gamma, Log logistics, and so on.). After that, it is necessary to estimate the underlying parameters.

2.3. Method of parameters estimation

The complete model includes a classical base rate and a vector B of dimension 3. If we choose a mechanical component based on the Weibull's model, then we have at least 6 parameters to be determined ($\beta, \eta, \gamma, b_1, b_2, b_3$), which is written as follows:

$$\lambda(t, z) = \frac{\beta}{\eta} \left(\frac{t - \gamma}{\eta} \right)^{\beta - 1} e^{b_1 z_1 + b_2 z_2 + b_3 z_3}$$

There are therefore two types of parameters, those belonging to the base model (Weibull for example) and those representing environmental conditions. The most preferable approach, is to proceed by step, starting with the evaluation of the vector B. Indeed, the estimation of Cox's model parameters can be done without prior knowledge on the base model of the failure rate $\lambda_0(t)$, this is an interesting feature to be exploited. To this end, we can use the partial Cox likelihood, or any other method based on the multiple regression. After that, it is possible to transform the lifetime data of the samples in the same environmental base, and proceeded to estimate the parameters of the base model with classical approach [BAG 95]. This approach will be developed in the application.

2.4. Notion of partial Cox's likelihood.

The contribution V_i of a component i , faulty at time t_i , having a partial likelihood V^* , is equal to the conditional probability that it is the element i subject to the constraints z_i which is faulty at time t_i , knowing the whole population at risk at the time t_i . This leads to the calculation defined below:

$$V_i = \frac{\lambda(t_i, z_i)}{\sum_{k \in n(t_i)} \lambda(t_i, z_k)}$$

$$V_i = \frac{\lambda(t_i, z_i)}{\sum_{k \in n(t_i)} \lambda(t_i, z_k)} = \frac{e^{b_i z_i}}{\sum_{k \in n(t_i)} e^{b_i z_k}}$$

$$V^* = \prod_{i=1}^n V_i$$

$$L^* = \ln(V^*)$$

$$L^* = \sum_{i=1}^n \left(b_i z_i - \ln \left(\sum_{j \in n(t_i)} e^{b_j z_j} \right) \right)$$

$$\frac{dL^*(\hat{b})}{db} = 0$$

3. Validation of modelling with simulation

We choose a base model representing a mechanical component with a failure mode of kind fatigue/corrosion. This is represented by $\beta=2,15$ and $\eta=12000$ hours. Environmental parameters represent the quality of design, the quality of process and environmental stress. We use Cox model for this representation, with: $b_1=1,5$, $b_2=0,5$, $b_3=1,7$, and $Z=[z_1^{n_1} z_2^{n_2} z_1^{n_3}]$, with $n_1=1, \dots, 3$, $n_2=n_3=1, \dots, 2$. The following codification was adopted: $z_1^1=0$; $z_1^2=1$; $z_1^3=2$; $z_2^1=0$; $z_2^2=1$; $z_3^1=0$; $z_3^2=1$.

The complete model is represented by following formula:

$$\lambda_{(t,z)} = \frac{2,15}{12000} \times \left(\frac{Ti}{12000} \right)^{1,15} \times e^{(2,15;0,5;1,7) \begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \end{pmatrix}}$$

The algorithm of simulation is based on the following steps.

1st. Stage: Creation of database of the lifetime of components in given environmental conditions.

$$T_{(i,0)} = 12000(-\ln NAH_i)^{1/2,15}, \quad T_{(i,z)} = \frac{T_{(i,0)}}{\left(e^{(2,15;0,5;1,7) \begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \end{pmatrix}} \right)^{1/2,15}}$$

2th Stage: Description of the database. From the simulation of 400 data we obtain a database of lifetime representing the duration of functioning of components. This allow to check the validity of reliability modelling method. This database is represented by the graph below.

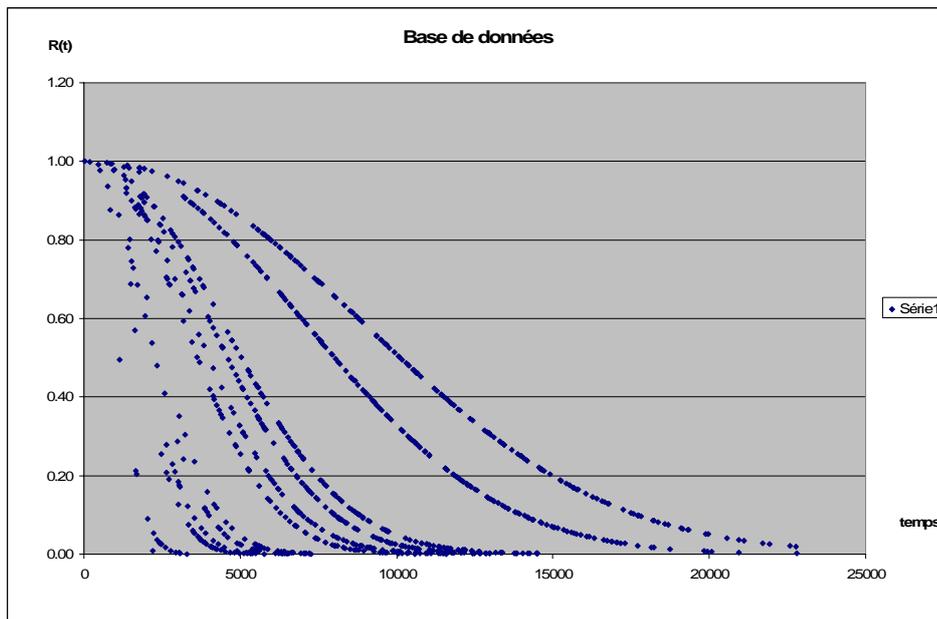


Figure 1. Simulation data

We take as reference case to find, the model with $z_1=z_2=z_3=1$, which led to the theoretical model:

$$R(t, z) = e^{-\left(\frac{t}{12000}\right)^{2.15} (1.5+0.5+1.7)}$$

The reference curve is in black on the graph (figure 2).

Rude modelling by a simple Weibull's model with 3 parameters. The estimate of parameters from the method of the maximum of likelihood, led to $\beta=1,22$, $\eta=4371$, $\gamma=0$. This model does not take into account parameters of environment.

Modelling from the complete Cox's model. The estimate of parameters is based on method of the maximum of likelihood and the partial likelihood of Cox, we obtained: $b_1=1,57$, $b_2=0,28$, $b_3=1,91$ and $\beta=2,28$, $\eta=12817$, $\gamma=0$. The complete model is represented by:

$$R(t, z) = e^{-\left(\frac{t}{12817}\right)^{2.29} (1.57Z_1+0.28Z_2+1.91Z_3)}$$

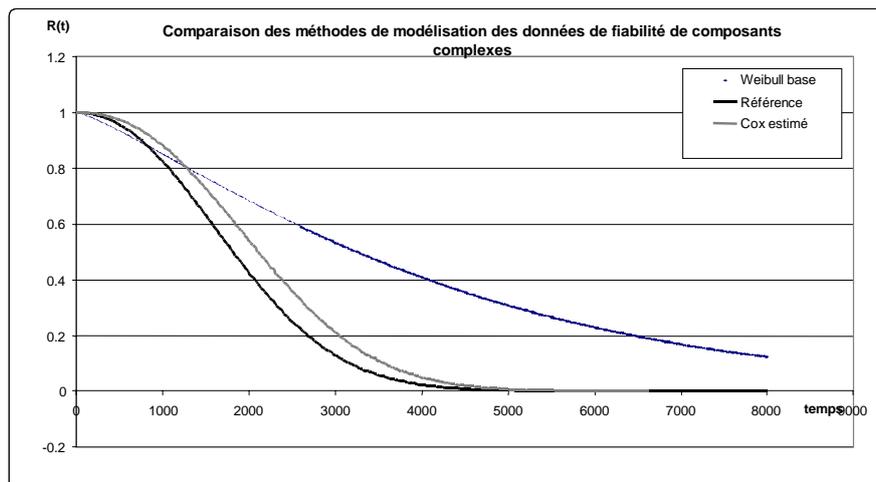


Figure 2. Comparaison of results

4. Conclusion

We can see on the graph a poor adequacy of the base model compared to the reference case. The Cox's model ameliorates adequacy significantly and gives advantage to be perfectly parametrized. We recommend the use of this model to have a good modelling of reliability of components, particularly when available information is sufficient to estimate parameters.

5. References

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